

# St. Patrick's Catholic Primary School

## Calculation Progression



This guidance lays out the expectations for both mental and written calculations for the 4 number operations and has been created to support the teaching of mathematics. This is underpinned by the use of models and images that support conceptual understanding and promotes a range of representations to be used across the primary years. Mathematical understanding is developed through use of representations that are first of all concrete (e.g. dienes apparatus and place value counters), and then pictorial (e.g. bar models) to then facilitate abstract working (e.g. standard written methods). This policy is a guide through an appropriate progression of representations and if at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate.

(This policy has been largely adapted from the white rose maths hub calculation policy with further material added. it is a working document and will be revised and amended as necessary.)

make addition  
total  add  
altogether sum  
more plus

**Early Learning Goal:**

*Using quantities and objects, children add two single-digit numbers and count on to find the answer.*

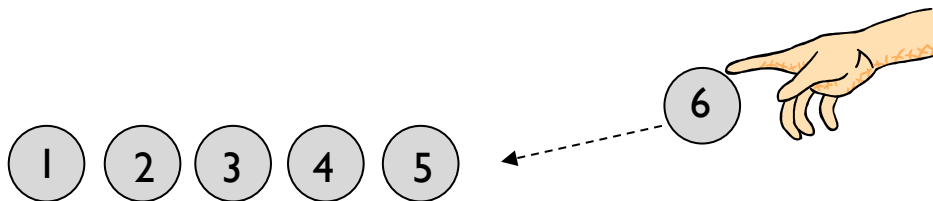
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

**Counting all method**

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating  $4 + 2$ , they are encouraged to count out four counters and count out two counters.



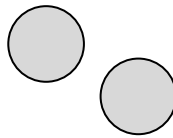
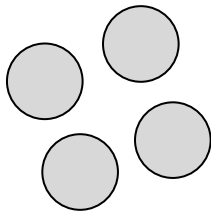
To find how many altogether, touch and drag them into a line one at a time whilst counting.



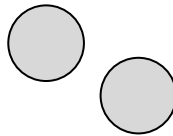
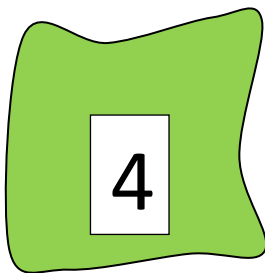
By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

### **Counting on method**

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating  $4 + 2$ , count out the two groups of counters as before.

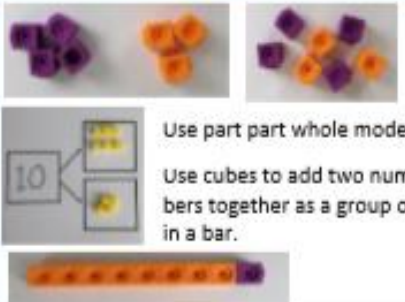
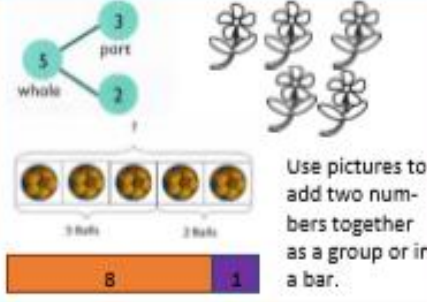
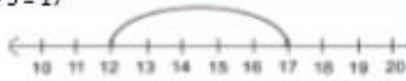
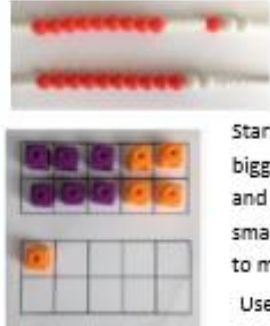
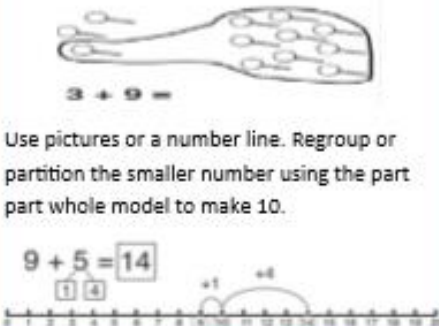
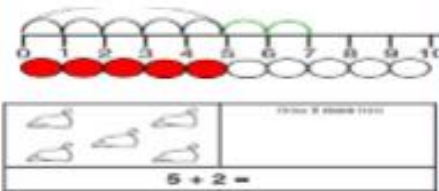


then cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before. **Those who are ready** may record their own calculations.

# Y1 ADDITION +

| Objective & Strategy                                                                   | Concrete                                                                                                                                                                                                            | Pictorial                                                                                                                                                                                                                                                                | Abstract                                                                                                                                                                                                                          |
|----------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Combining two parts to make a whole: part- whole model                                 |  <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>                            |  <p>Use pictures to add two numbers together as a group or in a bar.</p>                                                                                                               | <p><math>4 + 3 = 7</math></p>  <p><math>10 = 6 + 4</math></p> <p>Use the part-part whole diagram as shown above to move into the abstract.</p> |
| Starting at the bigger number and counting on                                          |  <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>         | <p><math>12 + 5 = 17</math></p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>                                          | <p><math>5 + 12 = 17</math></p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>                                                                                                  |
| Regrouping to make 10.<br><i>This is an essential skill for column addition later.</i> |  <p><math>6 + 5 = 11</math></p> <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p> |  <p><math>3 + 9 =</math></p> <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p> <p><math>9 + 5 = 14</math></p> | <p><math>7 + 4 = 11</math></p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>                                                                                                        |
| Represent & use number bonds and related subtraction facts within 20                   |  <p>2 more than 5.</p>                                                                                                           |  <p><math>5 + 2 =</math></p>                                                                                                                                                         | <p>Emphasis should be on the language</p> <p>'1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.'</p> <p>'8 is 3 more than 5.'</p>                                                                                         |

## Y1

### **End of Year Objective:**

***Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).***

Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value, they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and ones.

For example, when adding 11 and 5, they can make the 11 using a ten rod and a one.

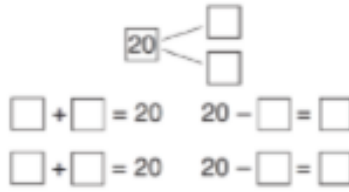
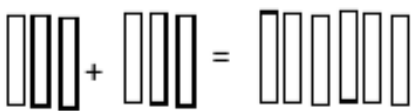
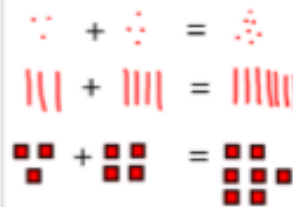
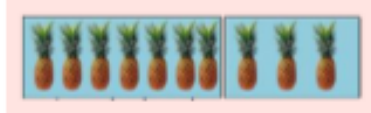
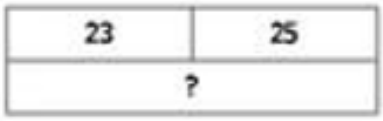


The ones can then be combined to aid with seeing the final total, e.g.

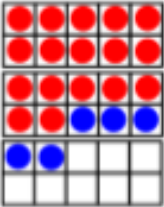
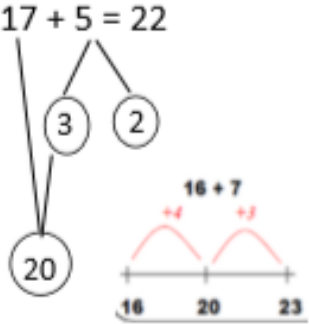
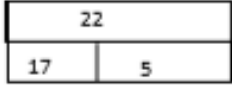
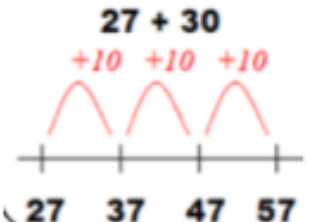
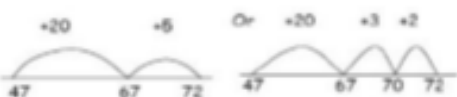
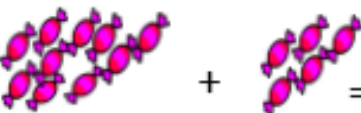


so  $11 + 5 = 16$ . If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

# Y2 ADDITION +

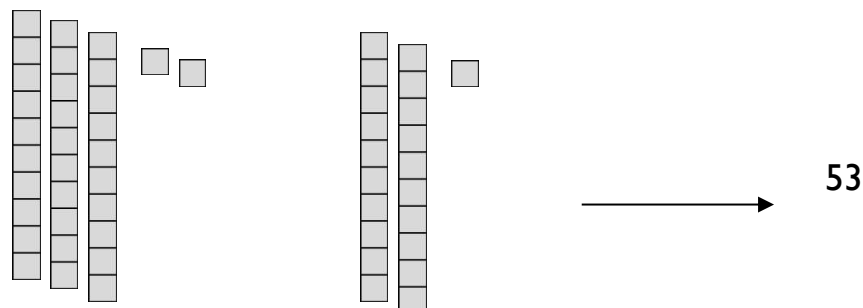
| Objective & Strategy                             | Concrete                                                                                                                                              | Pictorial                                                                                                                                                                                                                                    | Abstract                                                                                                |
|--------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| Adding multiples of ten                          | $50 = 30 + 20$<br><br>Model using dienes and bead strings            | <br>$3 \text{ tens} + 2 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$<br>$30 + 20 = \underline{\hspace{2cm}}$<br>Use representations for base ten. | $20 + 30 = 50$<br>$70 = 50 + 20$<br>$40 + \square = 60$                                                 |
| Use known number facts<br><b>Part part whole</b> |  Children explore ways of making numbers within 20                   | <br>$\square + \square = 20$ $20 - \square = \square$<br>$\square + \square = 20$ $20 - \square = \square$                                                 | $\square + 1 = 16$ $16 - 1 = \square$<br>$1 + \square = 16$ $16 - \square = 1$                          |
| Using known facts                                | $\square\square + \square\square = \square\square\square\square$<br> | <br>Children draw representations of H, T and O                                                                                                            | $3 + 4 = 7$<br><i>leads to</i><br>$30 + 40 = 70$<br><i>leads to</i><br>$300 + 400 = 700$                |
| Bar model                                        | <br>$3 + 4 = 7$                                                    | <br>$7 + 3 = 10$                                                                                                                                         | <br>$23 + 25 = 48$ |

# Y2 ADDITION +

| Objective & Strategy                                                                                     | Concrete                                                                                                                   | Pictorial                                                                                                                                                                                                                | Abstract                                                                                                                                                                                           |
|----------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Add a two digit number and ones<br><br>Children explore the pattern.<br>$17 + 5 = 22$<br>$27 + 5 = 32$   |  <p>Use ten frame to make 'magic ten'</p> | <p>Use part part whole and number line to model.</p>                                                                                  | $17 + 5 = 22$<br><br>Explore related facts<br>$17 + 5 = 22$<br>$5 + 17 = 22$<br>$22 - 17 = 5$<br>$22 - 5 = 17$  |
| Add a 2 digit number and tens<br><br>$25 + 10 = 35$<br>Explore that the ones digit does not change       |                                           |                                                                                                                                        | $27 + 10 = 37$<br>$27 + 20 = 47$<br>$27 + \square = 57$                                                                                                                                            |
| Add two 2-digit numbers<br><br>Model using dienes, place value counters and numicon                      |                                           |  <p>Use number line and bridge ten using part whole if necessary.</p>                                                                  | $25 + 47$<br>$20 + 40 = 60$<br>$5 + 7 = 12$<br>$60 + 12 = 72$                                                                                                                                      |
| Add three 1-digit numbers<br><br>Combine to make 10 first if possible, or bridge 10 then add third digit |                                         |  <p>Regroup and draw representation.</p>  $= 15$ | $4 + 7 + 6 = 10 + 7 = 17$<br><br>Combine the two numbers that make/ bridge ten then add on the third.                                                                                              |



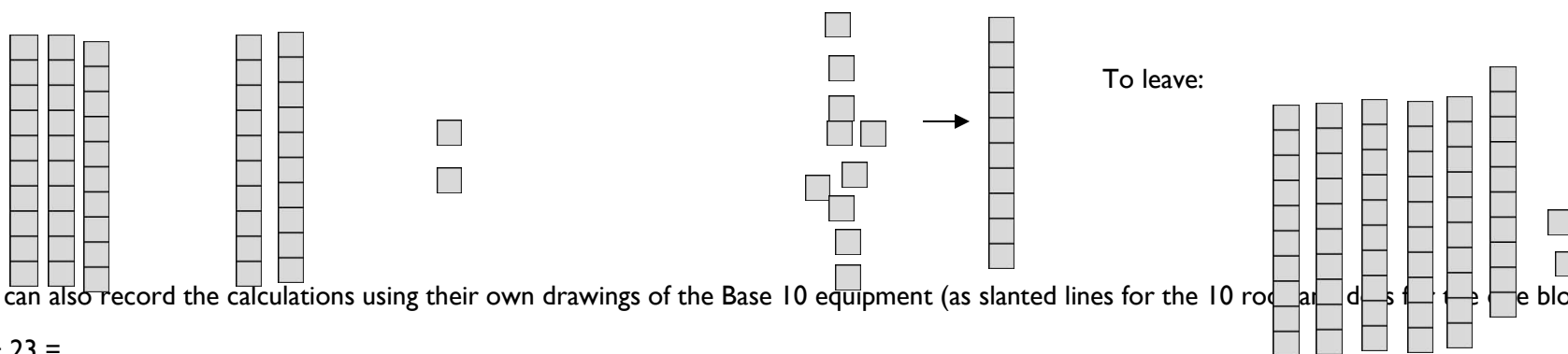
Children will continue to use the Base 10 equipment to support their calculations. For example, to calculate  $32 + 21$ , they can make the individual amounts, counting the tens first and then count on the ones.



When the ones total more than 10, children should be encouraged to exchange 10 ones/ones for 1 ten. This is the start of children understanding ‘carrying’ in vertical addition. For example, when calculating  $35 + 27$ , they can represent the amounts using Base 10 as shown:

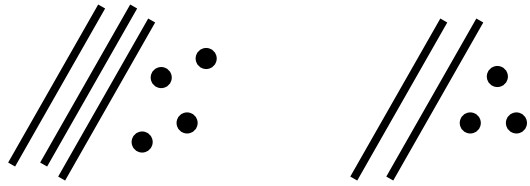


Then, identifying the fact that there are enough ones/ones to exchange for a ten, they can carry out this exchange:



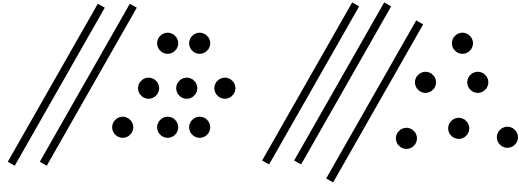
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the one blocks).

e.g.  $34 + 23 =$

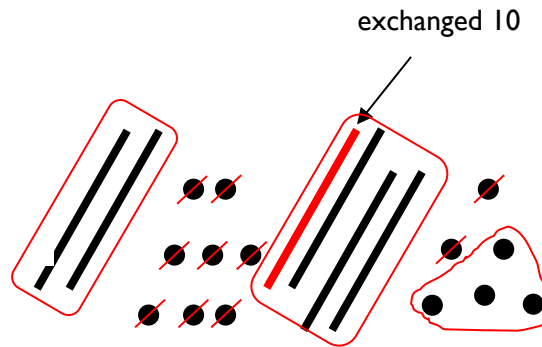


With exchange:

e.g.  $28 + 36 =$



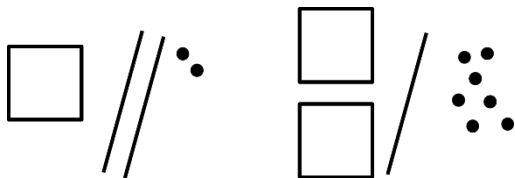
will become



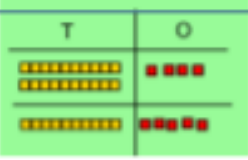
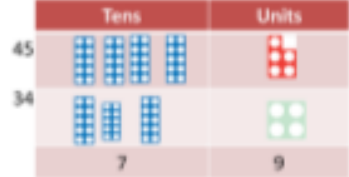
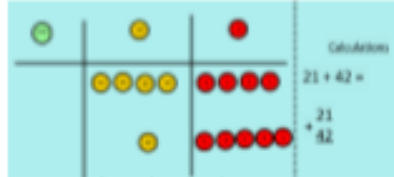
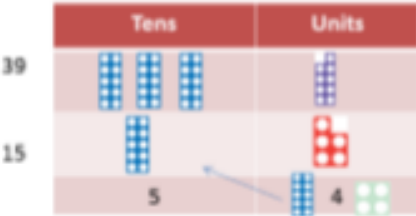
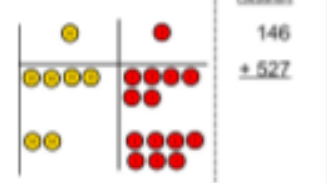
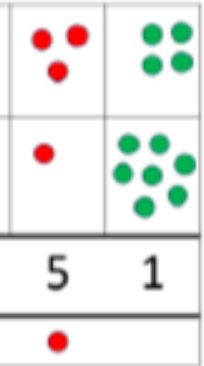
so  $28 + 36 = 64$

It is important that children circle the remaining tens and ones after exchange to identify the amount remaining.

This method can also be used with adding three digit numbers, e.g.  $122 + 217$  using a square as the representation of 100.



# Y3 ADDITION +

| Objective & Strategy                                                                                  | Concrete                                                                                                                                                                                                                                                                                                                                                                                | Pictorial                                                                                                                                                                                                                     | Abstract                                                                                                                                                                                                           |
|-------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3-digit numbers.</p> |  <p>Model using Dienes or numicon</p> <p>Add together the ones first, then the tens.</p>   <p>Move to using place value counters</p> | <p>Children move to drawing the counters using a tens and one frame.</p>                                                                    | $\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>                                                                                             |
| <p>Column Addition with regrouping.</p>                                                               |  <p>Exchange ten ones for a ten. Model using numicon and pv counters.</p>                                                                                                                                          |  <p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p> | $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$ |

### Y3

**End of Year Objective: Add numbers with up to three digits, using expanded written methods of columnar addition.**

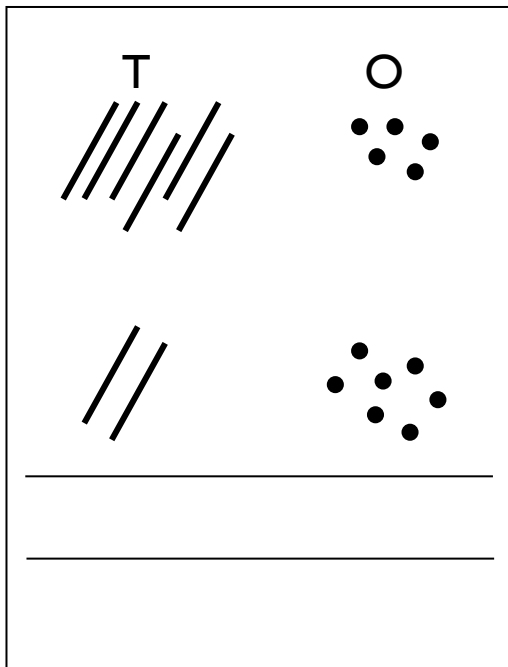
*If children are ready, then they can move on to the formal written method but this must not be rushed. It is expected that only the most able of GDS children will learn the formal written method prior to Year 4*

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

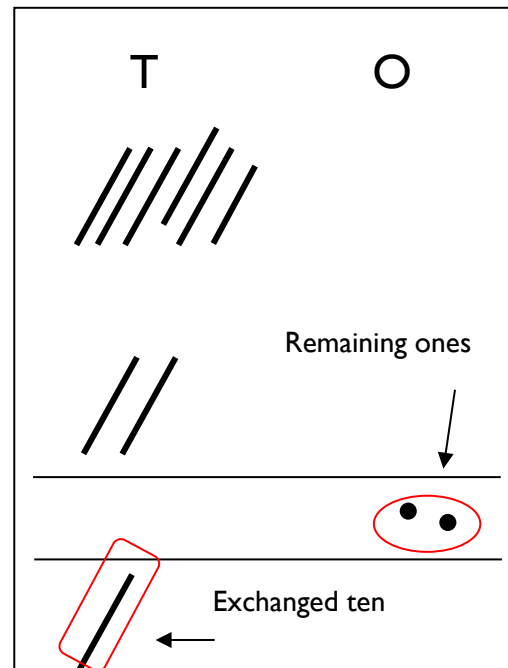
Children should add the **least significant digits** first (i.e. start with the ones/ones), and in an identical method to that from year 2, should identify whether there are greater than ten ones which can be exchanged for one ten.

They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns

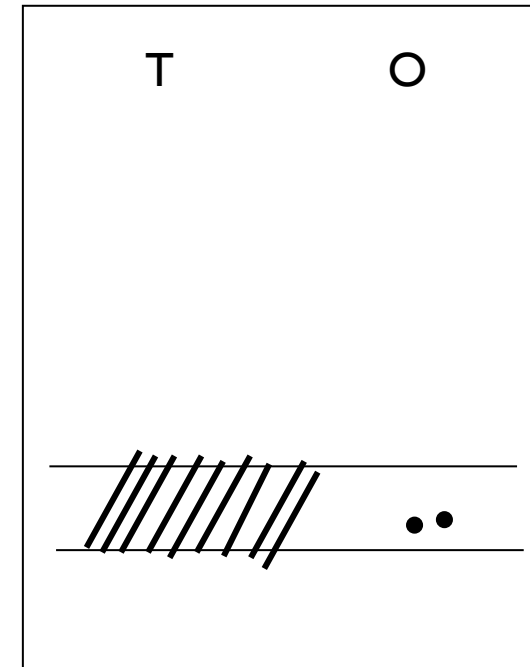
Step 1



Step 2



Step 3



### Written method

$$65 + 27$$

60 and 5

+ 20 and 7

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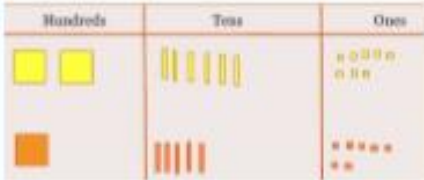
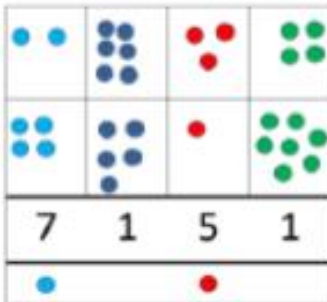
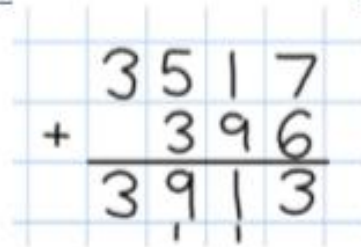
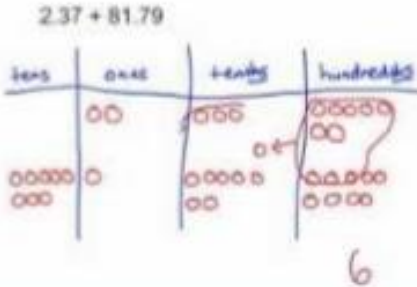
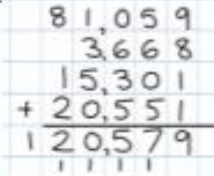
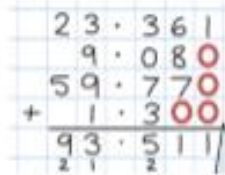
90 and 2

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10

Children would exchange ten ones for a ten, placing the exchanged ten below the equals sign. Any remaining ones that cannot be exchanged for a ten move into the equals sign as they are the ones part of the answer (as in the diagram in Step 2 above). If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 above).

Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially. This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method. By the end of year 3, children should also extend this method for three digit numbers.

| Objective & Strategy                                                                                                                                 | Concrete                                                                                                                                                                                                                                  | Pictorial                                                                                                                     | Abstract                                                                                                                                                                                                          |
|------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Y4—add numbers with up to 4 digits</p>                                                                                                            | <p>Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p>  |  <p>Draw representations using pv grid.</p> |  <p>Continue from previous work to carry hundreds as well as tens.</p> <p>Relate to money and measures.</p>                    |
| <p>Y5—add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>                                           | <p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>                                                         | <p>2.37 + 81.79</p>                         | <p>72.8</p> <p>+ 54.6</p> <p>127.4</p> <p>11</p>                                                                               |
| <p>Y6—add several numbers of increasing complexity</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p> | <p>As Y5</p>                                                                                                                                                                                                                              | <p>As Y5</p>                                                                                                                  |  <p>Insert zeros for place holders.</p>  |

# Y4-6 ADDITION +

## Y4

### End of Year Objective:

**Add numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar addition where appropriate.**

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

## Y5

### End of Year Objective:

**Add whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar addition).**

Children should continue to use the carrying method to solve calculations such as:

$$\begin{array}{r} 3364 \\ + 247 \\ \hline 3611 \\ \hline \end{array}$$

$$\begin{array}{r} 3121 \\ 37 \\ + 148 \\ \hline 3306 \\ \hline \end{array}$$

$$\begin{array}{r} 3.56 \\ + 2.47 \\ \hline 6.03 \\ \hline \end{array}$$

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

## Y6

### End of Year Objective:

**Add whole numbers and decimals using formal written methods (columnar addition).**

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline \end{array}$$

$$\begin{array}{r} 401.20 \\ 26.85 \\ + 0.71 \\ \hline 428.76 \\ \hline \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one one to another.



subtraction

subtract

take away

fewer



minus

less

difference between

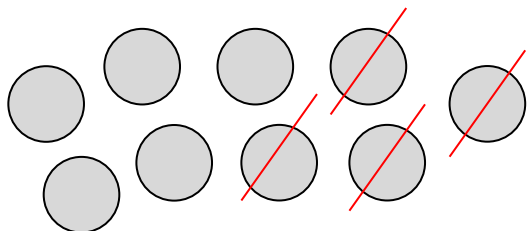
**Early Learning Goal:**

*Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.*

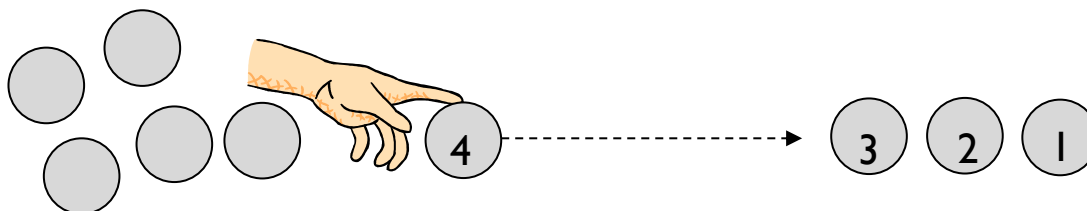
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

**Taking away**

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g.  $9 - 4$ .

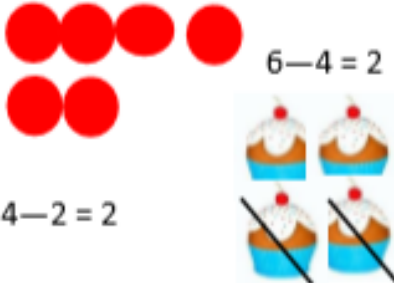
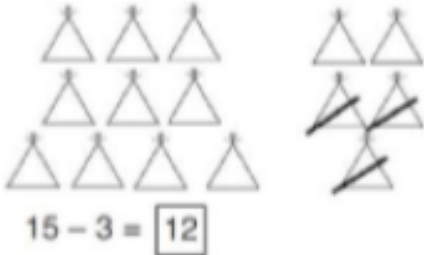
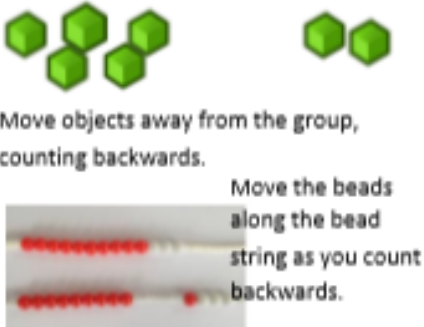
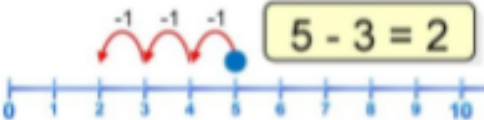
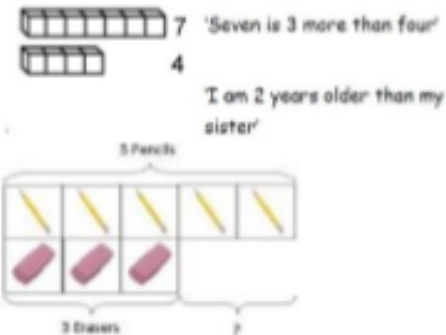
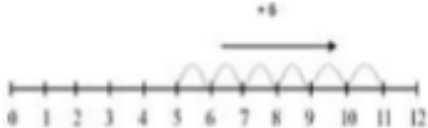


For illustration purposes, the amount being taken away are shown crossed out. Children would be encouraged to physically remove these using touch counting.

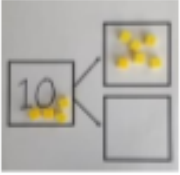
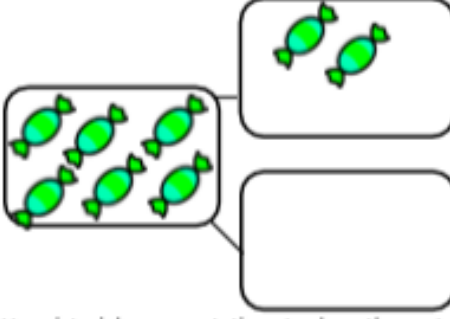
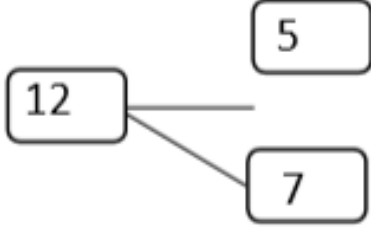
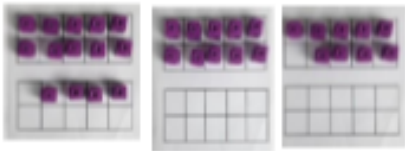


By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

**Those who are ready** may record their own calculations.

| Objective & Strategy | Concrete                                                                                                                                                                                                                              | Pictorial                                                                                                                                                                           | Abstract                                                                                           |
|----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| Taking away ones.    | <p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p><math>6 - 4 = 2</math></p> <p><math>4 - 2 = 2</math></p> |  <p><math>15 - 3 = 12</math></p> <p>Cross out drawn objects to show what has been taken away.</p> | <p><math>7 - 4 = 3</math></p> <p><math>16 - 9 = 7</math></p>                                       |
| Counting back        |  <p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>                    |  <p><math>5 - 3 = 2</math></p> <p>Count back in ones using a number line.</p>                     | <p>Put 13 in your head, count back 4. What number are you at?</p>                                  |
| Find the Difference  | <p>Compare objects and amounts</p>  <p>Lay objects to represent bar model.</p>                                                                     | <p>Count on using a number line to find the difference.</p>                                     | <p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p> |

# Y1 SUBTRACTION -

| Objective & Strategy                                                                                       | Concrete                                                                                                                                                                                                                                 | Pictorial                                                                                                                                                               | Abstract                                                                                                                                      |
|------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part-Part Whole model</p> |  <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$ |  <p>Use pictorial representations to show the part.</p>                               | <p>Move to using numbers within the part whole model.</p>  |
| <p>Make 10</p>                                                                                             | <p>14—9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>                                | <p>13—7</p>  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p> | <p>16—8</p> <p>How many do we take off first to get to 10? How many left to take off?</p>                                                     |
| <p>Bar model</p>                                                                                           |  $5 - 2 = 3$                                                                                                                                           |                                                                                      |  $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$      |

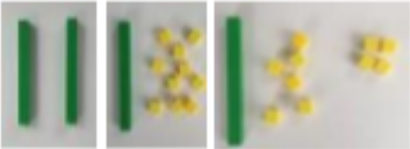
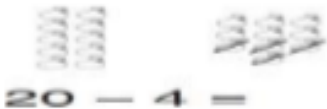
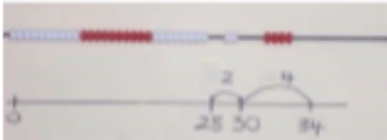
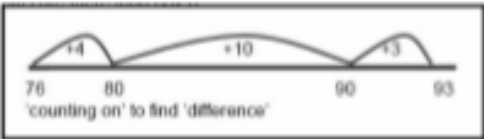
# Y1 SUBTRACTION -

## Y1

### **End of Year Objective:**

***Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).***

Children will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters, cubes and the ones from the Base 10 equipment, but not the tens, e.g.  $13 - 4$

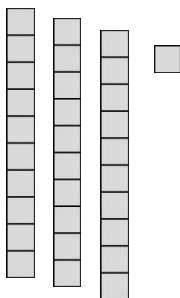
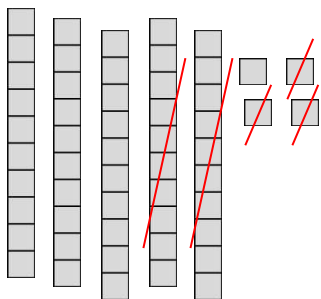
| Objective & Strategy                                                                                                   | Concrete                                                                                                                                                                                    | Pictorial                                                                                                                                                       | Abstract       |
|------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| Regroup a ten into ten ones                                                                                            |  <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>             |  $20 - 4 =$                                                                   | $20 - 4 = 16$  |
| Partitioning to subtract without regrouping.<br><i>'Friendly numbers'</i>                                              | $34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p> | <p>Children draw representations of Dienes and cross off.</p>  $43 - 21 = 22$ | $43 - 21 = 22$ |
| Make ten strategy<br><i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i> |  $34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>              |  <p>Use a number line to count on to next ten and then the rest.</p>         | $93 - 76 = 17$ |
|                                                                                                                        |                                                                                                                                                                                             |                                                                                                                                                                 |                |

# Y2 SUBTRACTION -

**End of Year Objective:**

**Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers.**

Children will begin to use the Base 10 equipment to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate  $54 - 23$ , children would count out 54 using the Base 10 equipment (5 tens and 4 ones). They need to consider whether there are enough ones/ones to remove 3, in this case there are, so they would remove 3 ones and then two tens, counting up the answer of 3 tens and 1 one to give 31.



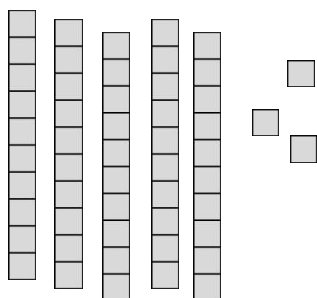
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the one blocks), e.g. to calculate  $39 - 17$  children would draw 39 as 3 tens (lines) and 4 ones (dots) and would cross out 7 ones and then one ten, counting up the answer of 2 tens and 2 ones to give 22.



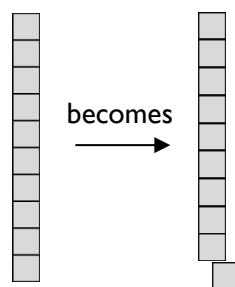
Circling the tens and ones that remain will help children to identify how many remain.

When the amount of ones to be subtracted is greater than the ones in the original number, an exchange method is required. This relies on children's understanding of ten ones being an equivalent amount to one ten. To calculate  $53 - 26$ , by using practical equipment, they would count out 53 using the tens and ones, as in Step 1. They need to consider whether there are enough ones/ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1



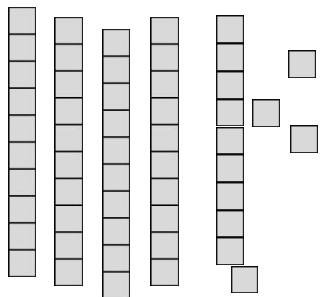
Step 2



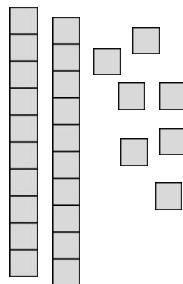
The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.



Step 3

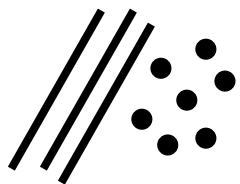


Step 4

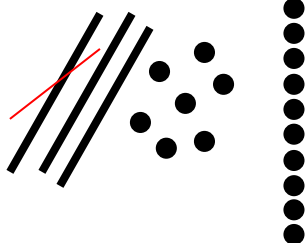


When recording their own drawings, when calculating  $37 - 19$ , children would cross out a ten and exchange for ten ones. Drawing them in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the ones that were already in place.

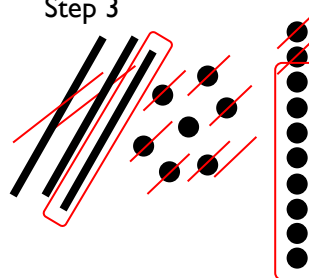
Step 1



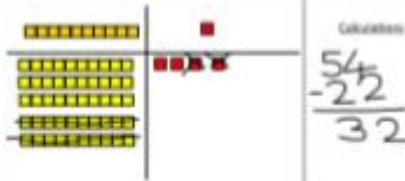
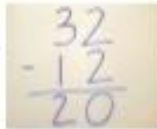
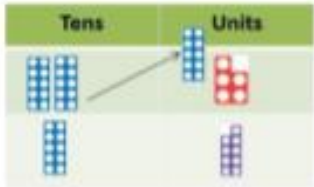
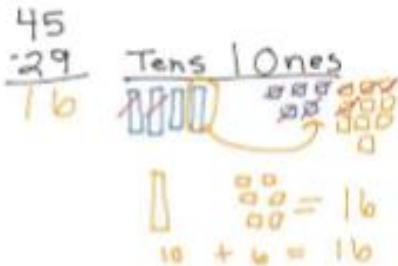
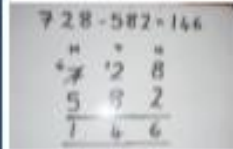
Step 2



Step 3



Circling the tens and ones that remain will help children to identify how many remain.

| Objective & Strategy                                     | Concrete                                                                                                                                                                                                                             | Pictorial                                                                                                                                      | Abstract                                                                                                                                                                                                                                     |
|----------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Column subtraction without regrouping (friendly numbers) |  $47 - 32$<br><br>Use base 10 or Numicon to model                  | <br>Draw representations to support understanding            | $47 - 24 = 23$ $\begin{array}{r} 47 \\ - 24 \\ \hline 23 \end{array}$ <p>intermediate step may be needed to lead to clear subtraction understanding.</p>  |
| Column subtraction with regrouping                       | <br>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange. | <br>Children may draw base ten or PV counters and cross off. | <br>Begin by partitioning into columns<br>by pv                                                                                                           |
|                                                          |                                                                                                                                                                                                                                      |                                                                                                                                                | <br>Then move to formal method.                                                                                                                           |

# Y3 SUBTRACTION -

### Y3

#### End of Year Objective:

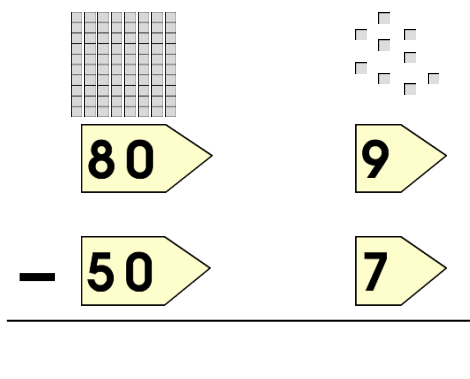
**Subtract numbers with up to three digits, using formal written method of columnar subtraction.\***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

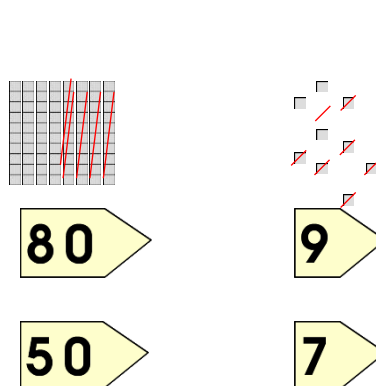
**It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.**

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the ones and tens as appropriate (as with the more informal method in Y2).

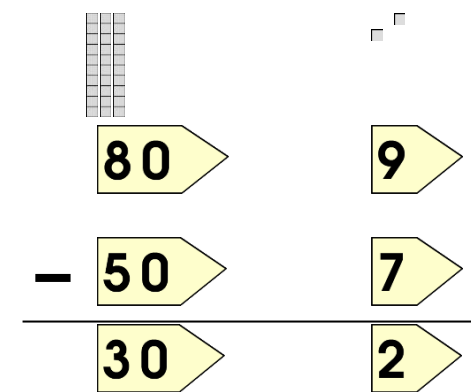
Step 1



Step 2



Step 3

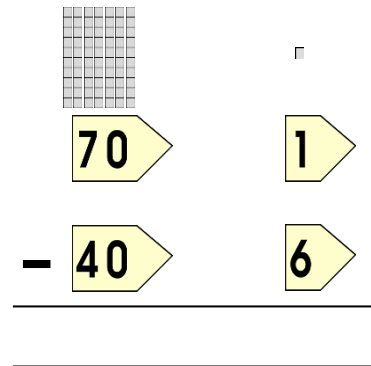


This will be recorded by the children as:

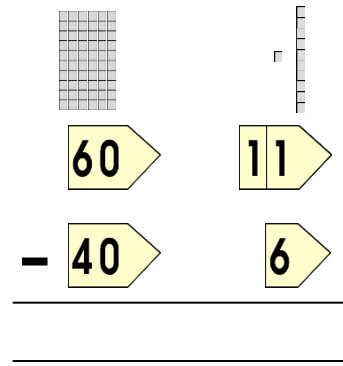
$$\begin{array}{rcl}
 80 & \text{and} & 9 \\
 - 50 & \text{and} & 7 \\
 \hline
 30 & \text{and} & 2 = 32
 \end{array}$$

From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough ones/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for Year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same ( $71 = 70 + 1 = 60 + 11$ ).

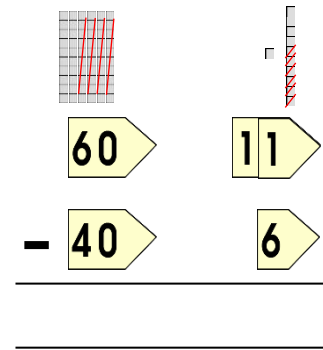
Step 1



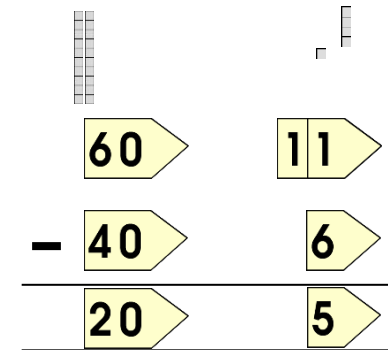
Step 2



Step 3



Step 4



$$\begin{array}{r}
 60 \\
 70 \text{ and } 11 \\
 -40 \text{ and } 6 \\
 \hline
 20 \text{ and } 5
 \end{array}
 = 25$$

By the end of year 3, children should also extend this method for three digit numbers.

## Y4

### End of Year Objective:

**Subtract numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar subtraction where appropriate.**

Children will move to Y4 using whichever method they were using as they transitioned from Y3.

Step 1

Step 2 (exchanging from tens to ones)

Step 3 (exchanging from hundreds to tens)

$$\begin{array}{r} 700 \text{ and } 50 \text{ and } 4 \\ - 200 \text{ and } 80 \text{ and } 6 \\ \hline \end{array}$$

$$\begin{array}{r} 40 \\ 700 \text{ and } \cancel{50} \text{ and } 14 \\ - 200 \text{ and } 80 \text{ and } 6 \\ \hline \end{array}$$

$$\begin{array}{r} 600 \quad 140 \\ \cancel{700} \text{ and } \cancel{50} \text{ and } 14 \\ - 200 \text{ and } 80 \text{ and } 6 \\ \hline \end{array}$$

$$\begin{array}{r} 600 \quad 140 \\ \cancel{700} \text{ and } \cancel{50} \text{ and } 14 \\ - 200 \text{ and } 80 \text{ and } 6 \\ \hline 400 \text{ and } 60 \text{ and } 8 = 468 \end{array}$$

#When children are ready, this leads on to the compact method of decomposition:

$$\begin{array}{r} \quad 6 \quad 14 \\ 4 \cancel{7} \cancel{5} 14 \\ - 3 \quad 2 \quad 8 \quad 6 \\ \hline 1 \quad 4 \quad 6 \quad 8 \end{array}$$

By the end of Y4, children should be using the written method confidently and with understanding.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

## Y5

### End of Year Objective:

**Subtract whole numbers with more than 4 digits and decimals with two decimal places, including formal written methods.**

Children should continue to use the decomposition method to solve calculations such as:

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one one to another

## Y6

### End of Year Objective:

**Subtract whole numbers and decimals using formal written methods (columnar subtraction).**

$$\begin{array}{r} \overset{5}{6} \overset{13}{4} \overset{13}{3} \overset{2}{2} \\ - \quad 4 \quad 6 \quad 8 \quad 1 \\ \hline 1 \quad 7 \quad 5 \quad 1 \end{array}$$

$$\begin{array}{r} \overset{3}{4} \overset{6}{1} \overset{11}{7} \overset{11}{2} \overset{10}{0} \\ - \quad 3 \quad 4 \quad . \quad 7 \quad 1 \\ \hline 3 \quad 8 \quad 2 \quad . \quad 4 \quad 9 \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one one to another.

multiplication

times

multiply

lots of

groups of



multiples of

## YR

### **Early Learning Goal:**

***Children solve problems, including doubling.***

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.

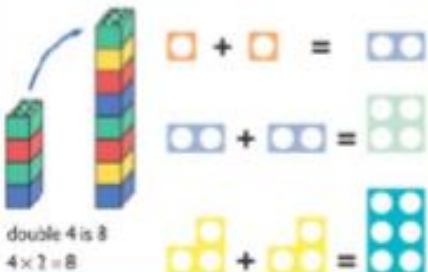
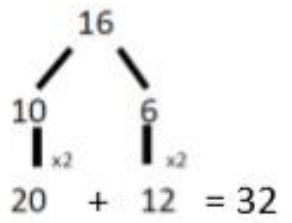
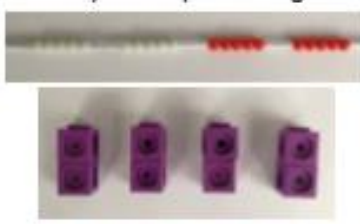
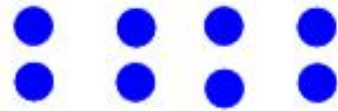
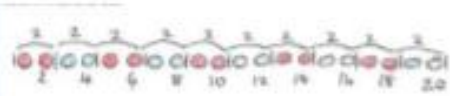


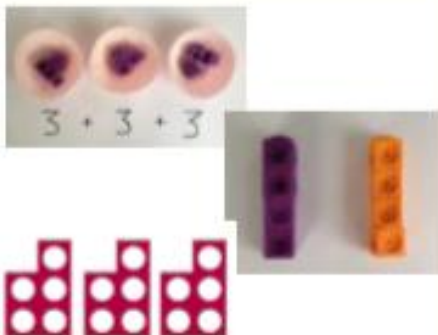
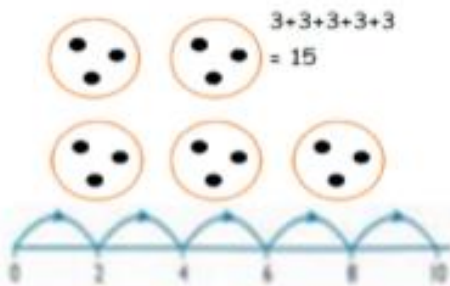
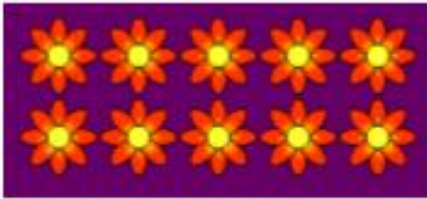
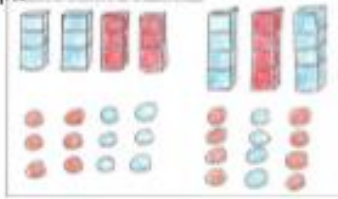
A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.





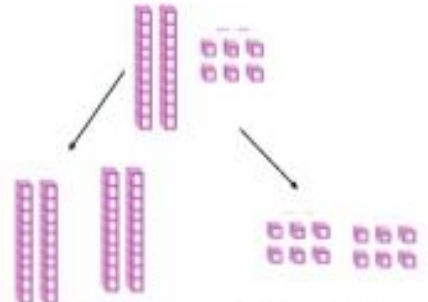
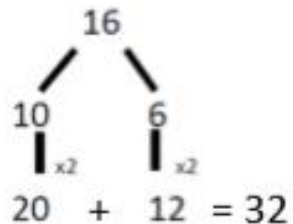
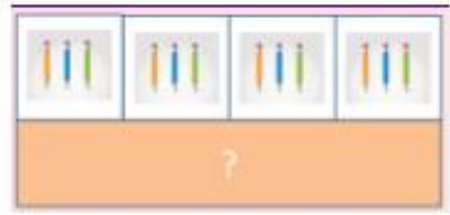
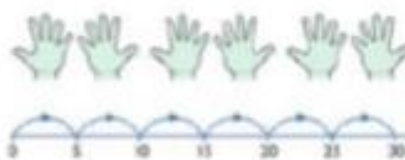
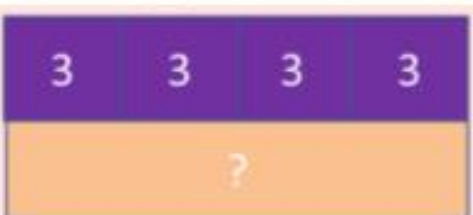
| Objective & Strategy                       | Concrete                                                                                                                                                                                                                                       | Pictorial                                                                                                                                                                                                                                  | Abstract                                                                                                                                                                                                      |
|--------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Doubling                                   | <p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8<br/><math>4 \times 2 = 8</math></p> | <p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p>                                                                                  | <p>Partition a number and then double each part before recombining it back together.</p>  <p><math>20 + 12 = 32</math></p> |
| Counting in multiples                      | <p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p>                                             |  <p>Children make representations to show counting in multiples.</p>  | <p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>                                                             |
| Making equal groups and counting the total |  <p>Use manipulatives to create equal groups.</p> <p><math>\square \times \square = 8</math></p>                                                            | <p>Draw  to show <math>2 \times 3 = 6</math></p> <p>Draw and make representations</p>                                                                 | <p><math>2 \times 4 = 8</math></p>                                                                                                                                                                            |

| Objective & Strategy | Concrete                                                                                                                                                                  | Pictorial                                                                                                                                                                                                                                       | Abstract                                                                                                                                                                       |
|----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Repeated addition    |  <p>Use different objects to add equal groups</p>                                        | <p>Use pictorial including number lines to solve prob. There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p>  $3 + 3 + 3 + 3 + 3 = 15$ | <p>Write addition sentences to describe objects and pictures.</p>  $2 + 2 + 2 + 2 + 2 = 10$ |
| Understanding arrays | <p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p>  | <p>Draw representations of arrays to show understanding.</p>                                                                                                 | $3 \times 2 = 6$ $2 \times 5 = 10$                                                                                                                                             |
|                      |                                                                                                                                                                           |                                                                                                                                                                                                                                                 |                                                                                                                                                                                |

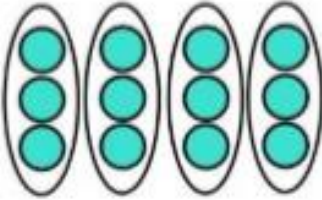
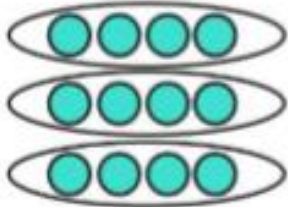
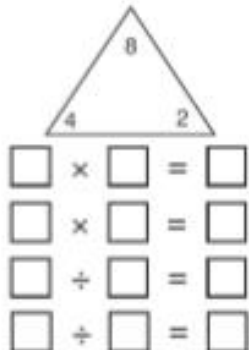
**End of Year Objective:**

**Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as how many eggs would be needed to fill an egg box.

| Objective & Strategy                                                  | Concrete                                                                                                                                                                                                                                                                                                                                    | Pictorial                                                                                                                                                                                                                                                                                     | Abstract                                                                                                                                                                                                                                                                                                         |
|-----------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Doubling                                                              | <p>Model doubling using dienes and PV counters.</p>  $40 + 12 = 52$                                                                                                                                                                                        | <p>Draw pictures and representations to show how to double numbers</p>                                                                                                                                                                                                                        | <p>Partition a number and then double each part before recombining it back together.</p>  $16 \begin{matrix} \swarrow & \searrow \\ 10 & 6 \\ \downarrow \times 2 & \downarrow \times 2 \\ 20 & 12 \\ + & \\ 32 \end{matrix}$ |
| Counting in multiples of 2, 3, 4, 5, 10 from 0<br>(repeated addition) | <p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$  | <p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>   | <p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> $4 \times 3 = \square$                                                                                                         |



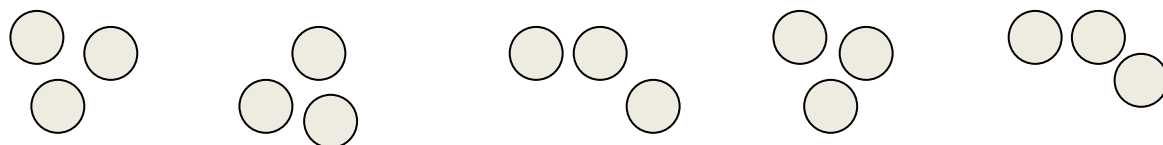
| Objective & Strategy                                                                                                                 | Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Pictorial                                                                                                                                                                                                                                                              | Abstract                                                                                                                                                                                                                                                                                                                                                                                                 |
|--------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Multiplication is commutative</p>                                                                                                 | <p>Create arrays using counters and cubes and Numicon.</p>    <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p>   | <p>Use representations of arrays to show different calculations and explore commutativity.</p>   | <p><math>12 = 3 \times 4</math></p> <p><math>12 = 4 \times 3</math></p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p><math>5 + 5 + 5 = 15</math></p> <p><math>3 + 3 + 3 + 3 + 3 = 15</math></p> <p><math>5 \times 3 = 15</math></p> <p><math>3 \times 5 = 15</math></p> |
| <p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                    | <p><math>2 \times 4 = 8</math></p> <p><math>4 \times 2 = 8</math></p> <p><math>8 \div 2 = 4</math></p> <p><math>8 \div 4 = 2</math></p> <p><math>8 = 2 \times 4</math></p> <p><math>8 = 4 \times 2</math></p> <p><math>2 = 8 \div 4</math></p> <p><math>4 = 8 \div 2</math></p> <p>Show all 8 related fact family sentences.</p>                                                                         |

**End of Year Objective:**

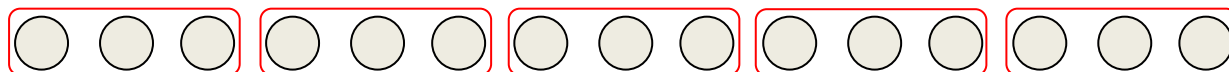
**Calculate mathematical statements for multiplication (*using repeated addition*) and write them using the multiplication (x) and equals (=) signs.**

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

$5 \times 3$  can be shown as five groups of three with counters, either grouped in a random pattern, as below:

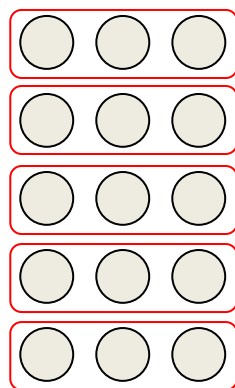


or in a more ordered pattern, with the groups of three indicated by the border outline:

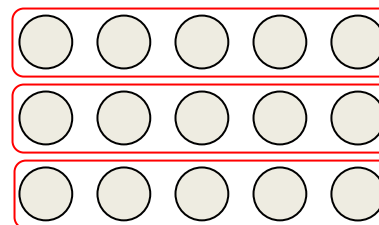


Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

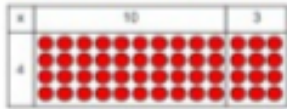
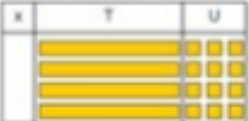
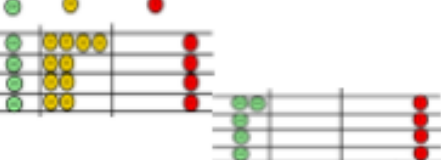
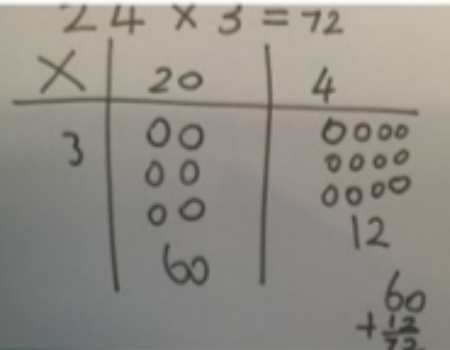
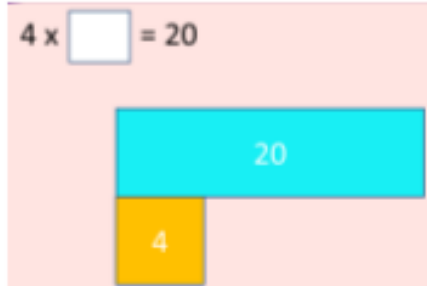
$5 \times 3^*$  can be represented as an array in two forms (as it has commutativity):



$$3 + 3 + 3 + 3 + 3 = 15$$



$$5 + 5 + 5 = 15$$

| Objective & Strategy | Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Pictorial                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Abstract                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |   |    |   |   |     |    |  |    |   |    |     |    |   |    |    |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|---|---|-----|----|--|----|---|----|-----|----|---|----|----|
| Grid method          | <p>Show the links with arrays to first introduce the grid method</p>  <p>4 rows of 10<br/>4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations<br/><math>4 \times 126</math></p> <p>Fill each row with 126</p>  <p>Calculations<br/><math>4 \times 126</math></p> <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer.</p> | <p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p>  | <p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1"><tr><td>X</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> <p><math>210 + 35 = 245</math></p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1"><tr><td></td><td>10</td><td>8</td></tr><tr><td>10</td><td>100</td><td>80</td></tr><tr><td>3</td><td>30</td><td>24</td></tr></table> | X | 30 | 5 | 7 | 210 | 35 |  | 10 | 8 | 10 | 100 | 80 | 3 | 30 | 24 |
| X                    | 30                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |    |   |   |     |    |  |    |   |    |     |    |   |    |    |
| 7                    | 210                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 35                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |    |   |   |     |    |  |    |   |    |     |    |   |    |    |
|                      | 10                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |    |   |   |     |    |  |    |   |    |     |    |   |    |    |
| 10                   | 100                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 80                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |    |   |   |     |    |  |    |   |    |     |    |   |    |    |
| 3                    | 30                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 24                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |    |   |   |     |    |  |    |   |    |     |    |   |    |    |

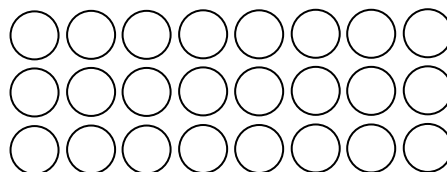
**End of Year Objective:**

**Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.\***

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$3 \times 8$$

They may show this using practical equipment:



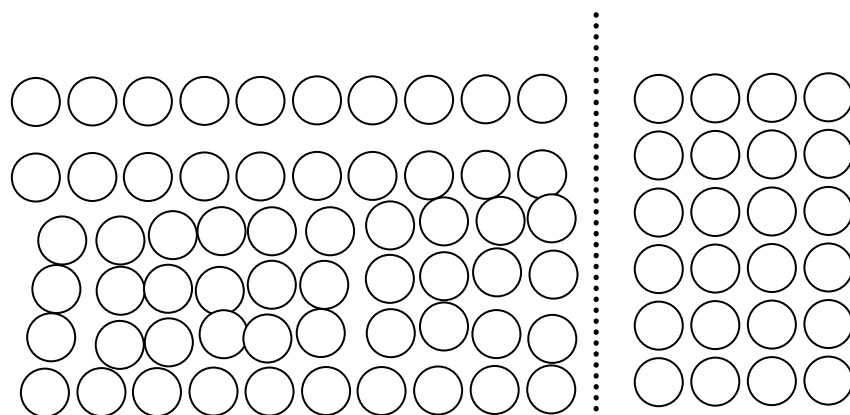
$$3 \times 8 = 8 + 8 + 8 = 24$$

or by jottings using squared paper:

|  |   |   |   |   |   |   |   |   |  |
|--|---|---|---|---|---|---|---|---|--|
|  | x | x | x | x | x | x | x | x |  |
|  | x | x | x | x | x | x | x | x |  |
|  | x | x | x | x | x | x | x | x |  |
|  |   |   |   |   |   |   |   |   |  |

$$3 \times 8 = 8 + 8 + 8 = 24$$

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and ones/ones to help them.<sup>10</sup> For example, when calculating  $14 \times 6$ , children should set out the array, then partition the array so that one array has ten columns and the other four.





Partitioning in this way, allows children to identify that the first array shows  $10 \times 6$  and the second array shows  $4 \times 6$ . These can then be added to calculate the answer:

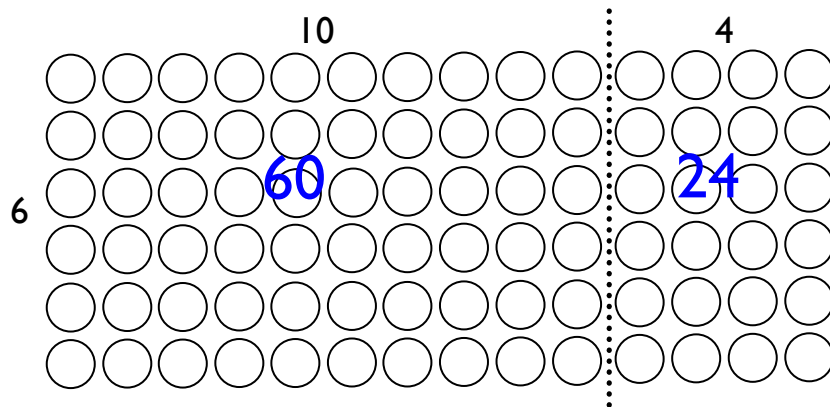
$$(6 \times 10) + (6 \times 4)$$

$$= 60 + 24$$

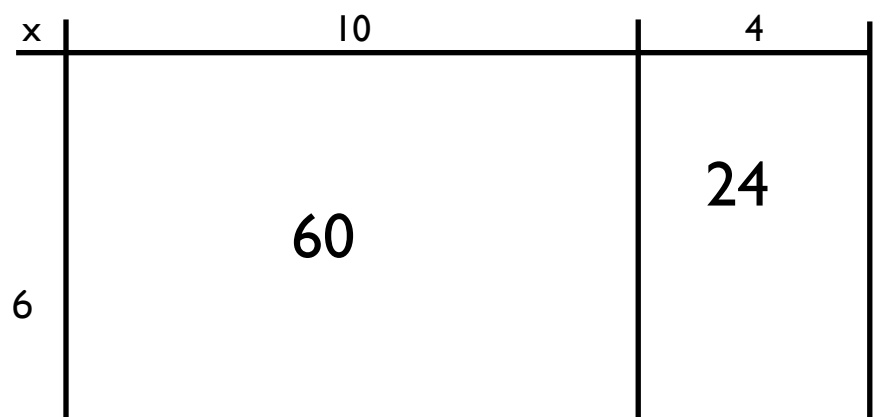
$$= 84$$

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

|      |    |    |
|------|----|----|
| 13 x |    |    |
| 8    |    |    |
| x    | 10 | 3  |
| 8    | 80 | 24 |

|      |
|------|
| 80   |
| + 24 |
| 104  |

also

|        |     |    |
|--------|-----|----|
| 37 x 6 |     |    |
| x      | 30  | 7  |
| 6      | 180 | 42 |

|      |
|------|
| 180  |
| + 42 |
| 222  |

Children should be using this method to solve problems and multiply numbers in the context of money or measures.

| Objective & Strategy                                                                                                                    | Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Pictorial                                                                                                                                                                                                                                                   | Abstract                                                                                                                                                                                                                                   |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
|-----------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|----|---|---|-----|----|---|---|---|---|---|---|---|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|-----|----|---|---|------|----|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|-----|---|---|---|---|---|
| <p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p> | <p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p> <p>Calculations<br/><math>4 \times 126</math></p> <p>Fill each row with 126</p> <p>Add up each col. making any exchanges needed</p>                                                                                                                                                                                                                                                                                                             | <p>Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below</p> | <p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> <p><math>210 + 35 = 245</math></p> | x    | 30 | 5 | 7 | 210 | 35 |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| x                                                                                                                                       | 30                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 5                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| 7                                                                                                                                       | 210                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 35                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| Column multiplication                                                                                                                   | <p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. <math>321 \times 2 = 642</math></p> <table><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr></table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p> | Hundreds                                                                                                                                                                                                                                                    | Tens                                                                                                                                                                                                                                       | Ones | 3  | 2 | 1 | 3   | 2  | 1 | 3 | 2 | 1 | 3 | 2 | 1 | <table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> <p>The grid method may be used to show how this relates to a formal written method.</p> | x | 300 | 20 | 7 | 4 | 1200 | 80 | 28 | <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p> <table><tr><td>x</td><td>327</td><td>4</td></tr><tr><td>1</td><td>3</td><td>0</td><td>8</td></tr></table> <p>This may lead to a compact method.</p> | x | 327 | 4 | 1 | 3 | 0 | 8 |
| Hundreds                                                                                                                                | Tens                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Ones                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| 3                                                                                                                                       | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 1                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| 3                                                                                                                                       | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 1                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| 3                                                                                                                                       | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 1                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| 3                                                                                                                                       | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 1                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| x                                                                                                                                       | 300                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 20                                                                                                                                                                                                                                                          | 7                                                                                                                                                                                                                                          |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| 4                                                                                                                                       | 1200                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 80                                                                                                                                                                                                                                                          | 28                                                                                                                                                                                                                                         |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| x                                                                                                                                       | 327                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 4                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                            |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |
| 1                                                                                                                                       | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 0                                                                                                                                                                                                                                                           | 8                                                                                                                                                                                                                                          |      |    |   |   |     |    |   |   |   |   |   |   |   |                                                                                                                                                                                                                   |   |     |    |   |   |      |    |    |                                                                                                                                                                                                                                                                                                |   |     |   |   |   |   |   |

## Y4

### End of Year Objective:

**Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.**

Children will move to Y4 using whichever method they were using as they transitioned from Y3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

$$79 \times 8$$

|   |     |    |
|---|-----|----|
| x | 70  | 9  |
| 8 | 560 | 72 |

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \end{array}$$

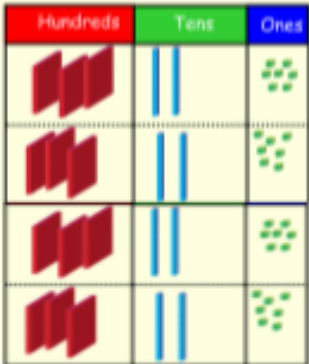
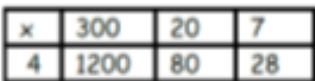
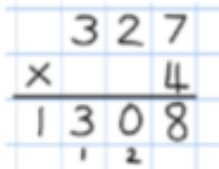
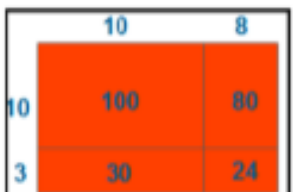
To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that  $7 \times 8 = 56$  to know that  $70 \times 8 = 560$ .

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

$$346 \times 8$$

|   |      |     |    |
|---|------|-----|----|
| x | 300  | 40  | 6  |
| 8 | 2400 | 320 | 48 |

$$\begin{array}{r} 2400 \\ + 320 \\ + 48 \\ \hline 2768 \end{array}$$

| Objective & Strategy                                | Concrete                                                                                                                                                                                                                                                                                                                                                | Pictorial                                                                                                                                                               | Abstract                                                                                                                                                                                                                                                                                                                                                                               |
|-----------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Column Multiplication for 3 and 4 digits x 1 digit. |  <p>It is important at this stage that they always multiply the ones first.</p> <p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. <math>321 \times 2 = 642</math></p> |                                                                                      | $\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$   <p>This will lead to a compact method.</p>                                                                            |
| Column multiplication                               | <p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>                                                                                                                                                                                                                                                   |   |  <p>18 x 3 on the first row<br/>(8 x 3 = 24, carrying the 2 for 20, then 1 x 3)</p> <p>18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first</p> $\begin{array}{r} 1234 \\ \times 6 \\ \hline 7404 \\ 12340 \\ \hline 19744 \end{array}$ <p>(1234 x 6)<br/>(1234 x 10)</p> |

Continue to use bar modelling to support problem solving

## Y5

### End of Year Objective:

**Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.**

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

$$4346 \times 8$$

|   |        |      |     |    |
|---|--------|------|-----|----|
| x | 4 000  | 300  | 40  | 6  |
| 8 | 32 000 | 2400 | 320 | 48 |

$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g.

$$2693 \times 24$$

|    |       |       |      |    |
|----|-------|-------|------|----|
| x  | 2000  | 600   | 90   | 3  |
| 20 | 40000 | 12000 | 1800 | 60 |
| 4  | 8000  | 2400  | 360  | 12 |

$$\begin{array}{r} 40000 \\ + 8000 \\ + 12000 \\ + 2400 \\ + 1800 \\ + 360 \\ + \end{array}$$

## Y6

### End of Year Objective:

**Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.**

By the end of Y6, children should be able to use the grid method to multiply any number by a two-digit number. They should also develop the method to be able to multiply decimal numbers with up to two decimal places, e.g.

$$4.92 \times 3$$

|   |    |     |      |
|---|----|-----|------|
| x | 4  | 0.9 | 0.02 |
| 3 | 12 | 2.7 | 0.06 |

$$\begin{array}{r} 12 \\ + 2.7 \\ + 0.06 \\ \hline 14.76 \end{array}$$

A handwritten grid method calculation for  $3.19 \times 8$ . The grid is 4 columns by 3 rows. The top row contains '3', '.', '1', '9'. The second row contains 'x', '8', and two empty cells. A horizontal line is drawn after the second row. The bottom row contains the result '25', '.', '5', '2'. Below the grid, there are small handwritten numbers '1' and '7' under the first and third columns respectively, likely representing the decimal places of the factors.

division  
share       divided by  
divide       divided into  
equal groups of



**Early Learning Goal:**

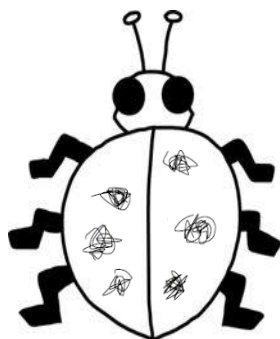
***Children solve problems, including halving and sharing.***

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



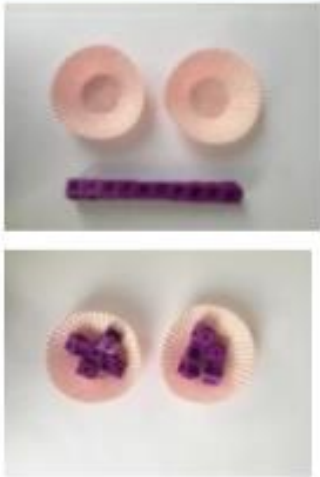
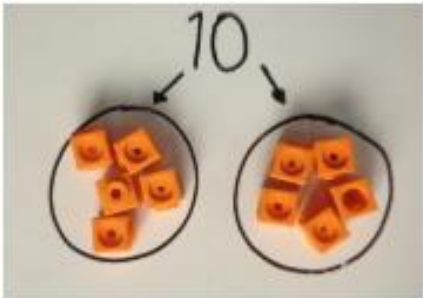
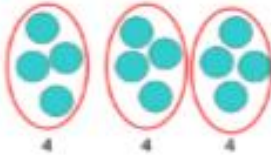
They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



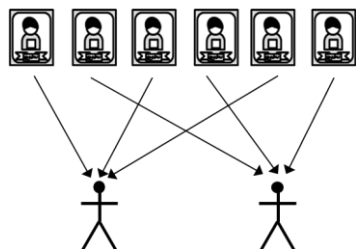
A child's jotting showing how they shared the apples at snack time between two groups.

| Objective & Strategy                                                   | Concrete                                                                                                                                                                                                                                    | Pictorial                                                                                                                                                                                                                                                                                                            | Abstract                               |
|------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------|
| <p>Division as sharing</p> <p><i>Use Gordon ITPs for modelling</i></p> | <br><br> <p>I have 10 cubes, can you share them equally in 2 groups?</p> | <p>Children use pictures or shapes to share quantities.</p>  <p>8 shared between 2 is 4</p><br><p>Sharing:</p>  <p>12 shared between 3 is 4</p> | <p>12 shared between 3 is</p> <p>4</p> |

**End of Year Objective:**

**Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.

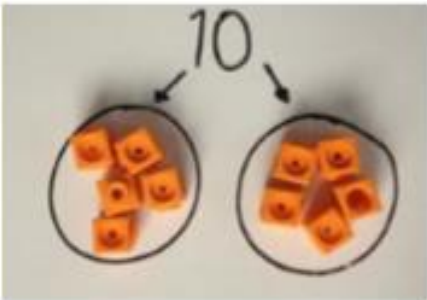
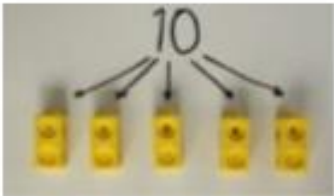
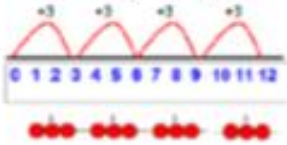


Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '... left over'.

# Y2

# DIVISION

# ÷

| Objective & Strategy | Concrete                                                                                                                                                                                                                                                                                             | Pictorial                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Abstract                                                                    |
|----------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| Division as sharing  |  <p>I have 10 cubes, can you share them equally in 2 groups?</p>                                                                                                                                                    | <p>Children use pictures or shapes to share quantities.</p>  <p><math>8 \div 2 = 4</math></p> <p>Children use bar modelling to show and support understanding.</p>  <p><math>12 \div 4 = 3</math></p>                                                                                         | $12 \div 3 = 4$                                                             |
| Division as grouping | <p>Divide quantities into equal groups.</p> <p>Use cubes, counters, objects or place value counters to aid understanding.</p>   | <p>Use number lines for grouping</p>  <p><math>12 \div 3 = 4</math></p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p><math>20 \div 5 = ?</math><br/> <math>5 \times ? = 20</math></p> | $28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p> |

**End of Year Objective:**

**Calculate mathematical statements for division within the multiplication tables and write them using the division ( $\div$ ) and equals ( $=$ ) signs.**

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

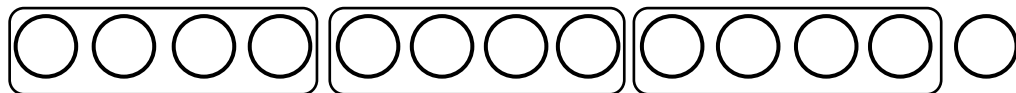
$$12 \div 3 =$$



Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

They should also continue to develop their knowledge of division with remainders, e.g.

$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ remainder } 1$$

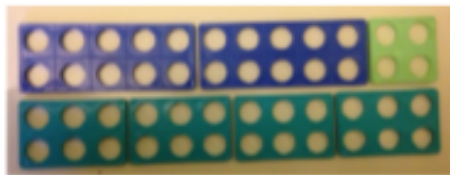
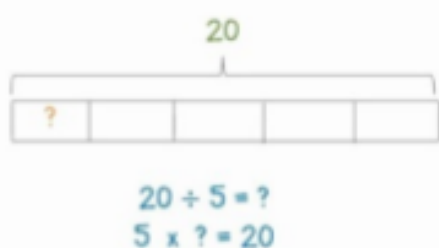
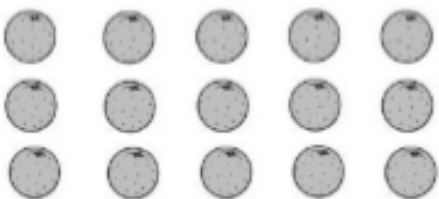
Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation  $13 \div 4$ , the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?

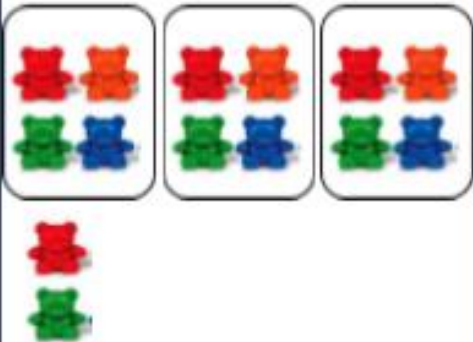
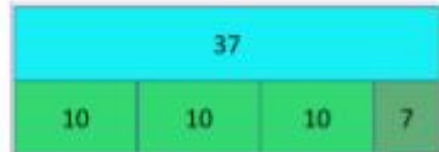
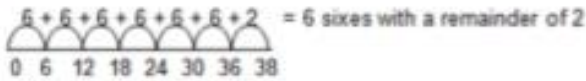
Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still needs to be placed into a box)

| Objective & Strategy | Concrete                                                                                                                                                                                                                                                                                                                                        | Pictorial                                                                                                                                                                                           | Abstract                                                                                                                                                                                                                                                                                                                                                                               |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Division as grouping | <p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$                                    | <p>Continue to use bar modelling to aid solving division problems.</p>  <p>20 ÷ 5 = ?<br/>5 × ? = 20</p>          | <p>How many groups of 6 in 24?</p> $24 \div 6 = 4$                                                                                                                                                                                                                                                                                                                                     |
| Division with arrays |  <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg <math>15 \div 3 = 5</math>    <math>5 \times 3 = 15</math><br/> <math>15 \div 5 = 3</math>    <math>3 \times 5 = 15</math></p> | <p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p>  | <p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> <p><math>7 \times 4 = 28</math><br/> <math>4 \times 7 = 28</math><br/> <math>28 \div 7 = 4</math><br/> <math>28 \div 4 = 7</math><br/> <math>28 = 7 \times 4</math><br/> <math>28 = 4 \times 7</math><br/> <math>4 = 28 \div 7</math><br/> <math>7 = 28 \div 4</math></p> |



| Objective & Strategy      | Concrete                                                                                                                                                                             | Pictorial                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Abstract                                                                                                                                                                                               |
|---------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Division with remainders. | <p><math>14 \div 3 =</math></p> <p>Divide objects between groups and see how much is left over</p>  | <p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p>  <p><b>Example without remainder:</b><br/> <math>40 \div 5</math><br/>           Ask "How many 5s in 40?"<br/>  <math>5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 8 \text{ fives}</math></p> <p><b>Example with remainder:</b><br/> <math>38 \div 6</math><br/>  <math>6 + 6 + 6 + 6 + 6 + 6 + 2 = 6 \text{ sixes with a remainder of } 2</math></p> <p>For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.</p> | <p>Complete written divisions and show the remainder using r.</p> <p><math>29 \div 8 = 3 \text{ REMAINDER } 5</math></p> <p>↑   ↑   ↑   ↑<br/>           dividend   divisor   quotient   remainder</p> |



**End of Year Objective:**

**Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods \***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$43 \div 8 =$$



$$43 \div 8 = 5 \text{ remainder } 3$$

In preparation for developing the ‘chunking’ method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

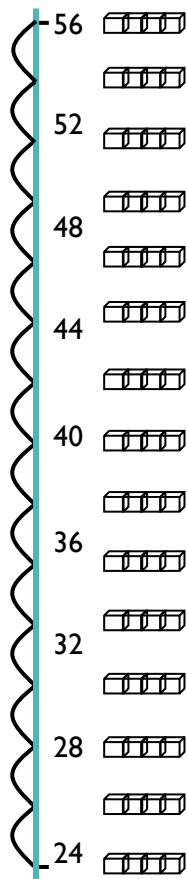
Stage 1 – repeatedly subtracting individual groups of the divisor

Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

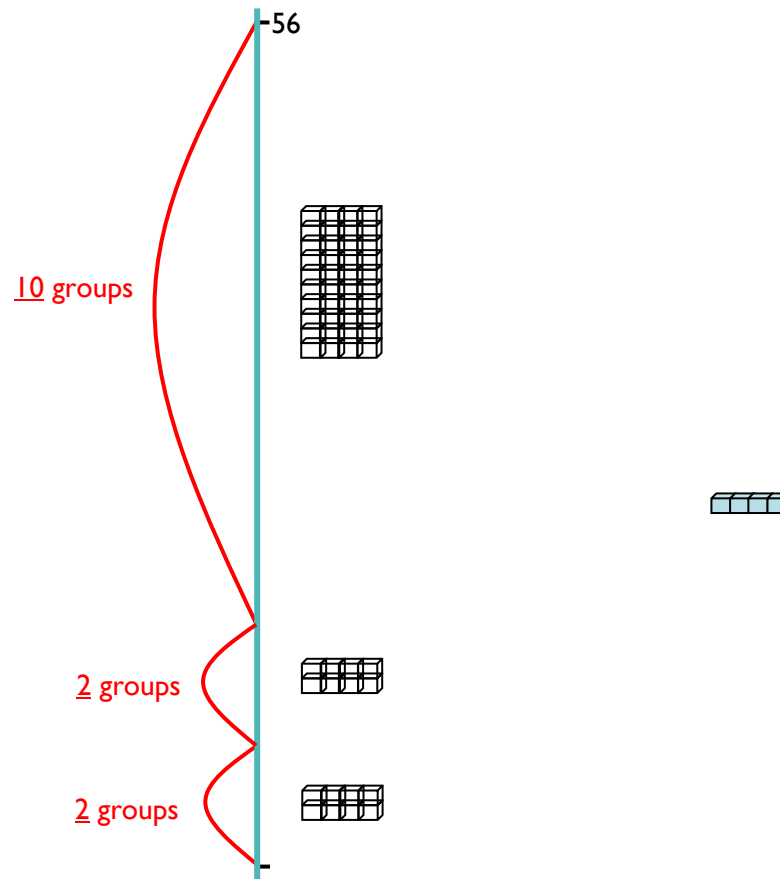
Stage 1

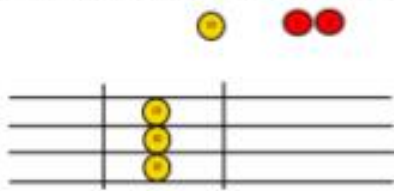
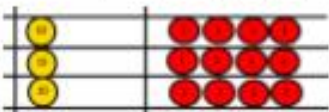
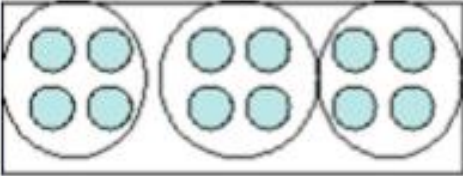
$$56 \div 4 = 14 \text{ (groups of 4)}$$



Stage 2

$$56 \div 4 = 10(\text{groups of } 4) + 2(\text{groups of } 4) + 2(\text{groups of } 4) \\ = 14(\text{groups of } 4)$$



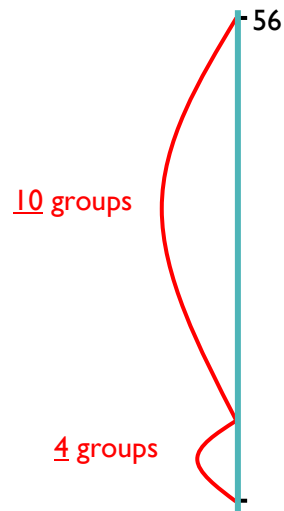
| Objective & Strategy                                              | Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Pictorial                                                                                                                                                                                                                                                                                         | Abstract                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|-------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Divide at least 3 digit numbers by 1 digit.<br><br>Short Division | <p>96 ÷ 3      Tens      Units</p> <p>                 3          2</p>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p>42 ÷ 3 =</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p> | <p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p> | <p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ $\begin{array}{r} 0.663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$ |

## Y4

### End of Year Objective:

*Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.*

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.



$$\begin{array}{r} 14 \\ 4 \overline{) 56} \\ \underline{40} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

Answer: 14

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

$$73 \div 3$$

$$\begin{array}{r}
 24\text{r}1 \\
 3 \overline{) 73} \\
 - 30 \\
 \hline
 43 \\
 - 30 \\
 \hline
 13 \\
 - 6 \\
 \hline
 7 \\
 - 6 \\
 \hline
 1
 \end{array}$$

Key facts box

|     |    |
|-----|----|
| 1x  | 3  |
| 2x  | 6  |
| 5x  | 15 |
| 10x | 30 |

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

$$196 \div 6$$

$$\begin{array}{r}
 32\text{r}4 \\
 6 \overline{) 196} \\
 - 120 \\
 \hline
 76 \\
 - 60 \\
 \hline
 16 \\
 - 12 \\
 \hline
 4
 \end{array}$$

Key facts box

|     |     |
|-----|-----|
| 1x  | 6   |
| 2x  | 12  |
| 4x  | 24  |
| 5x  | 30  |
| 10x | 60  |
| 20x | 120 |

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

## Y5

### End of Year Objective:

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. Any remainders should be shown as integers, e.g.

$$523 \div 8$$

$$\begin{array}{r} 65\text{r}3 \\ 8 \overline{) 523} \\ - 320 \quad 40\times \\ \hline 203 \\ - 160 \quad 20\times \\ \hline 43 \\ - 40 \quad 5\times \\ \hline 3 \end{array}$$

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

$$2458 \div 7$$

$$\begin{array}{r} 351\text{r}1 \\ 7 \overline{) 2458} \\ - 2100 \quad 300\times \\ \hline 358 \\ - 350 \quad 50\times \\ \hline 8 \\ - 7 \quad 1\times \\ \hline 1 \end{array}$$

## Y6

### End of Year Objective:

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

$$6367 \div 28$$

$$\begin{array}{r} 227r11 \\ 28 \overline{)6367} \\ \underline{-5600} \\ 767 \\ \underline{-560} \\ 207 \\ \underline{-140} \\ 67 \\ \underline{-56} \\ 11 \end{array} \quad \begin{array}{l} 200x \\ 20x \\ 5x \\ 2x \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places, e.g.  $362 \div 17$

$$362 \div 17$$

To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

$$\begin{array}{r}
 21.29 \\
 17 \overline{) 362} \\
 \underline{- 340} \phantom{00} \\
 22 \phantom{00} \\
 \underline{- 17} \phantom{00} \\
 5.0 \phantom{00} \\
 \underline{- 3.4} \phantom{00} \\
 1.60 \phantom{00} \\
 \underline{- 1.53} \phantom{00} \\
 0.07
 \end{array}$$

When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables,

e.g. if  $2 \times 17 = 34$ , I know that  $0.2 \times 17 = 3.4$

if  $9 \times 17 = 153$ ,  $0.9 \times 17 = 15.3$  so  $0.09 \times 17 = 1.53$

For simple fraction and decimal equivalents, this could also be demonstrated using a simple calculation such as  $13 \div 4$  to show the remainder initially as a fraction.



Using practical equipment, children can see that for  $13 \div 4$ , the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is  $\frac{1}{4}$ . To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).



$$3574 \div 8$$

$$\begin{array}{r}
 8 \overline{) 3574} \\
 \underline{3200} \quad 400x \\
 374 \quad 40x \\
 \underline{320} \quad 6x \\
 54 \\
 \underline{48} \\
 6
 \end{array}$$

$$\begin{array}{r}
 6 \quad \leftarrow \text{remainder} \\
 \underline{8} \quad \leftarrow \text{divisor}
 \end{array}$$

So  $3574 \div 8$  is  $446\frac{6}{8}$   
 (when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half:  $\frac{1}{2} = 0.5$

Quarters:  $\frac{1}{4} = 0.25$ ,  $\frac{3}{4} = 0.75$

Fifths:  $\frac{1}{5} = 0.2$ ,  $\frac{2}{5} = 0.4$ ,  $\frac{3}{5} = 0.6$ ,  $\frac{4}{5} = 0.8$

Tenths:  $\frac{1}{10} = 0.1$ ,  $\frac{2}{10} = 0.2$ ,  $\frac{3}{10} = 0.3$ ,  $\frac{4}{10} = 0.4$ ,  $\frac{5}{10} = 0.5$ ,  $\frac{6}{10} = 0.6$ ,  $\frac{7}{10} = 0.7$ ,  $\frac{8}{10} = 0.8$ ,  $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above,  $3574 \div 8$ , children should be able to identify that the remainder as a fraction of  $\frac{6}{8}$  can be written as  $\frac{3}{4}$  in its lowest terms. As  $\frac{3}{4}$  is equivalent to 0.75, the answer can therefore be written as 446.75

